Neutron EDM from Electric and Chromoelectric Dipole Moments of Quarks

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Using QCD sum rules, we calculate the electric dipole moment of the neutron d_n induced by all CP violating operators up to dimension five. We find that the chromoelectric dipole moments of quarks $\tilde{d}_i$, including that of the strange quark, provide significant contributions comparable in magnitude to those induced by the quark electric dipole moments d_i. When the theta term is removed via the Peccei-Quinn symmetry, the strange quark contribution is also suppressed and
$$d_n = (1 \pm 0.5)\left\{1.1e(\tilde{d}_d + 0.5\tilde{d}_u) + 1.4(d_d - 0.25d_u)\right\}.$$

Experimental discovery of CP violation in flavor-conserving channels would provide a clear indication of new physics around the electroweak scale. This stimulates continuing experimental efforts to detect electric dipole moments in elementary particles and heavy atoms [1]. The extraordinary precision, $d_n < 6 \times 10^{-26}$ *ecm* [2], obtained in measurements of the electric dipole moment (EDM) of the neutron, allows us to probe energy scales inaccessible in direct collider experiments. The information on fundamental CP violating parameters, such as CP-odd phases and masses of superpartners in supersymmetric (SUSY) models, is encoded in the coefficients of CP-odd effective operators $\mathcal{O}^{(i)}$, evolved down to 1 GeV. However, our ability to extract this information depends critically on the quality of calculations of the neutron EDM induced by these operators.

In this Letter, we report the results of the first systematic analysis of d_n, induced by all CP violating operators with dimension 4 and 5, within the QCD sum rule framework [3]. The complete set $\{\mathcal{O}^{(i)}\}$ of operators at the 1 GeV scale includes the theta term and the electric and chromoelectric dipole moments of quarks (EDMs and CEDMs):

$$\delta\mathcal{L} = -\sum_{q=u,d,s}\overline{q}(m_q + i\theta_q\gamma_5)q + \theta_G\frac{\alpha_s}{8\pi}G\tilde{G}$$
$$-\frac{i}{2}\sum_{q=u,d,s}d_q\overline{q}F\sigma\gamma_5 q - \frac{i}{2}\sum_{q=u,d,s}\tilde{d}_q\overline{q}g_s G\sigma\gamma_5 q, \quad (1)$$

where $F_{\mu\nu}$ and $G_{\mu\nu}$ are the electromagnetic and gluonic field strength tensors. With these sources, the neutron EDM can be written as a linear combination of the coefficients in Eq. (1),

$$d_n = d_n(\bar{\theta}) + d_n^{\text{EDM}}(d_u, d_d, d_s) + d_n^{\text{CEDM}}(\tilde{d}_u, \tilde{d}_d, \tilde{d}_s). \quad (2)$$

The first term $d_n(\bar{\theta})$ arises from the dimension four θ–term in (1), where $\bar{\theta} = \sum_q \theta_q + \theta_G$ is the physical combination. In the absence of e.g. the Peccei-Quinn (PQ) compensation mechanism [4], this is the most important source term, and the calculation of $d_n(\bar{\theta})$ within

QCD sum rules was performed in [5, 6]. Here we shall present the calculation of the dimension five contributions, $d_n^{\text{EDM}}(d_q)$ and $d_n^{\text{CEDM}}(\tilde{d}_q)$, which are required for phenomenological analyses. In fact, the results of $d_n^{\text{EDM}}(d_q)$ can be extracted from calculations of the tensor charges of light quarks in the proton [7]. At the same time, earlier calculations of $d_n^{\text{CEDM}}(\tilde{d}_q)$ in QCD sum rules [8] predict an unexpected suppression of CEDM contributions, contrasting with chiral loop estimates which indicate sizable effects [9]. Therefore, the main focus of this work is the crucial task of calculating the contributions of CEDMs, which also depend on the presence or absence of the PQ mechanism [10].

The starting point for the calculation is the correlator of currents $\eta_n(x)$ with quantum numbers of the neutron in a background with nonzero CP-odd sources and an electromagnetic field $F_{\mu\nu}$,

$$\Pi(Q^2) = i\int d^4x e^{ip\cdot x}\langle 0|T\{\eta_n(x)\overline{\eta}_n(0)\}|0\rangle_{\text{CP},F}, \quad (3)$$

where $Q^2 = -p^2$, with p the current momentum.

In the presence of CP violating sources it is necessary to take into account mixing between the neutron current and its CP conjugates. Thus we parametrize the interpolating current η_n in the form,

$$\eta_n = (j_1 + \beta j_2) + i\epsilon_{\text{CP}}(i_1 + \beta i_2), \quad (4)$$

where the two conventional neutron interpolators, $j_1 = 2\epsilon_{abc}(d_a^T C\gamma_5 u_b)d_c$ and $j_2 = 2\epsilon_{abc}(d_a^T C u_b)\gamma_5 d_c$, are combined with their CP conjugates, $i_1 = 2\epsilon_{abc}(d_a^T C u_b)d_c$ and $i_2 = 2\epsilon_{abc}(d_a^T \gamma_5 C u_b)\gamma_5 d_c$. The parameters ϵ and β in (4) play rather different roles. Firstly, ϵ reflects mixing induced by the CP violating source and will be calculated below. Thus to linear order in the sources,

$$\langle\eta_n\overline{\eta}_n\rangle = \langle j\overline{j}\rangle_{F,\text{CP}} + i\epsilon_{\text{CP}}\langle j\overline{i} + i\overline{j}\rangle_F + O(\epsilon_{\text{CP}})^2, \quad (5)$$

where,

$$\epsilon_{\text{CP}} = \frac{i}{2}\frac{\langle i_1\overline{j}_1 - j_1\overline{i}_1\rangle_{\text{CP}}}{\langle j_1\overline{j}_1 - i_1\overline{i}_1\rangle} \quad (6)$$

The second parameter β reflects the existence of the two interpolators j_1 and j_2, and would disappear from the result of an exact calculation. It can therefore be used to optimize the convergence of the operator product expansion (OPE) of (3). Within the sum rules formalism, one has the imperative of suppressing the contribution of excited states and higher dimensional operators in the OPE, and thus its convenient to choose β to this end. We shall therefore keep β arbitrary, and optimize once we have knowledge of the structure of the sum rule.

In analyzing the correlator (3) an additional consideration is that when CP-symmetry is broken by a generic quark-gluon CP-violating source, the coupling between the physical neutron state, described by a spinor v, and the current η_n acquires an additional phase factor,

$$\langle 0|\eta_n|N\rangle = \lambda e^{i\alpha\gamma_5/2} v, \qquad (7)$$

where the coupling naturally decomposes as $\lambda = \lambda_1 + \beta\lambda_2$ which in part motivates our parametrization above, with the same parameter β for the CP conjugate currents i_1 and i_2. With regard to electromagnetic form factors, the unphysical phase α in (7) can mix electric (d) and magnetic (μ) dipole moment structures and complicate the extraction of d from the sum rule. As discussed in [5], there is a unique tensor structure contributing to the neutron double pole term which is not contaminated by this phase, namely $\{F\sigma\gamma_5, \not{p}\}$. Therefore, it is this structure that we shall use to construct the EDM sum rule from (3). An additional advantage of using this structure comes from the absence of unknown vacuum tensor polarizabilities [7] in the calculation of $d_n^{\mathrm{EDM}}(d_q)$.

We now proceed to study the OPE associated with (3). To next-to-leading order in the OPE the relevant classes of diagrams we need to consider are shown in Fig. 1 ((a), (b) and (c)). Diagrams of the form (d), although suffering no loop factor suppression, are nonetheless suppressed due to combinatorial factors and the small numerical size of $(\langle\bar{q}q\rangle)^2$ relative to the natural scales in the problem. (This partially explains why the results of previous analyses [8] indicated a suppression of $d_n^{\mathrm{CEDM}}(\tilde{d}_q)$).

The vacuum structure is conveniently encoded in a generalized propagator expanded in the background field and the associated condensates. The condensates are then parametrized in terms of various susceptibilities defined as [11]:

$$\langle\bar{q}\sigma_{\mu\nu}q\rangle_F = \chi_q F_{\mu\nu}\langle\bar{q}q\rangle; \ \ g\langle\bar{q}(G_{\mu\nu}^a t^a)q\rangle_F = \kappa_q F_{\mu\nu}\langle\bar{q}q\rangle$$
$$g\langle\bar{q}G\sigma q\rangle = -m_0^2; \ \ 2g\langle\bar{q}\gamma_5(\tilde{G}_{\mu\nu}^a t^a)q\rangle_F = i\xi_q F_{\mu\nu}\langle\bar{q}q\rangle, \ \ (8)$$

and henceforth we follow [11] and assume that $\chi_q = \chi e_q$ etc., with flavor independent susceptibilities χ, κ, ξ. The dependence on CP violating parameters is either explicit, e.g. θ_q and d_q, or implicit in certain vacuum condensates in the case of $\tilde{d}_q$ and will be discussed shortly.

The relevant contributions to the OPE are exhibited in Fig. 1(a-c), but the explicit expressions are quite unwieldy, and we shall defer full details [12], and simply
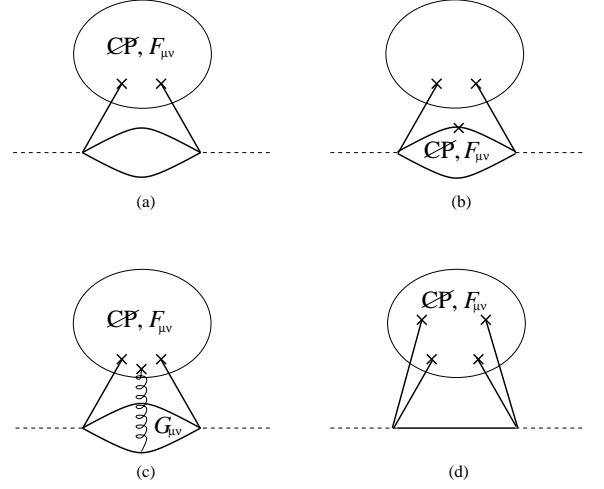

FIG. 1: Various classes of diagrams contributing to the CP-odd structure $\{F\sigma\gamma_5, \not{p}\}$.

present the resulting OPE expression. In momentum space we find

$$\Pi(Q^2) = -\frac{i\ln(-p^2)}{64\pi^2}\langle\bar{q}q\rangle\{F\sigma\gamma_5, \not{p}\}\left[\pi^{(\chi)} + \pi^{(\xi,\kappa)} + \pi^q\right]$$
$$+\frac{i}{16\pi^2 p^2}\ln\left(-\frac{\mu_{IR}^2}{p^2}\right)\langle\bar{q}q\rangle\{F\sigma\gamma_5, \not{p}\}\left[\pi_L^v + \pi_L^d\right], \quad (9)$$

corresponding to the first two nontrivial orders in the OPE. The relevant contributions from $\bar{\theta}$ and the CEDMs are contained in,

$$\pi^{(\chi)} = 4(1+\beta)^2\chi_d m_d P_d - (1+\beta)^2\chi_u m_u P_u$$
$$+2(1-\beta^2)m_*(\chi_u + \chi_d)(P_u - P_d), \qquad (10)$$

and

$$\pi^{(\xi,\kappa)} = \frac{1}{4}\tilde{d}_d\left[\alpha_d^+(3+2\beta+3\beta^2) - \alpha_u^+(1-\beta^2)\right.$$
$$\left.-\alpha_d^-(1-\beta)^2\right] + \frac{1}{8}\tilde{d}_u\left[2\alpha_d^+(1-\beta^2) - \alpha_u^+(1+\beta)^2\right], \quad (11)$$

where we have defined $P_q = \theta_q - i\langle\bar{q}\gamma_5 q\rangle_{\mathrm{CP}}/\langle\bar{q}q\rangle$, $m_* = m_u m_d/(m_u + m_d)$, and $\alpha^{\pm} = 2\kappa \pm \xi$ for concision. In obtaining this result it is crucial to take into account the mixing with the CP conjugate currents in (5). The parameters controlling the mixing are the unphysical phases $\theta_G - \theta_q$ and the quark mass difference $m_d - m_u$. Here, as expected, the addition of the mixing terms ensures the absence of the unphysical phases $\theta_G - \theta_q$.

The quark EDM contributions are

$$\pi^q = d_d\left[10 + 6\beta^2\right] - d_u\left[3 + 2\beta - \beta^2\right]. \qquad (12)$$

At subleading order, terms with a logarithmic momentum dependence arise and involve an infrared cutoff μ_{IR}. The relevant contributions are,

$$\pi_L^v = (1-\beta)^2 m_d e_d P_d$$
$$+(1-\beta^2)m_*(e_u + e_d)(P_d - P_u) \qquad (13)$$
$$\pi_L^d = \frac{1}{12}(1-\beta)^2 m_0^2(d_d - e_d\tilde{d}_d) \qquad (14)$$

The next problem to address is the calculation of the vacuum matrix elements P_u and P_d in Eq. (9). These terms require the evaluation of correlators of the form $\int d^4y \langle \overline{q}\gamma_5 q(x), i\delta\mathcal{L}(y)\rangle$, where $\delta\mathcal{L}$, given in Eq. (1), involves in particular the θ-term and the color EDM sources which may be extracted from the vacuum at leading order in the background electromagnetic field. The case of θ was discussed at length in [5, 13, 14]. Here we shall concentrate on the CEDMs and evaluate these correlators in chiral perturbation theory, saturating them by π_0 and η_8 exchange, assuming the decoupling of a heavy singlet state, and incorporating the effect of the chiral anomaly [15]. We then obtain,

$$m_{u(d)}P_{u(d)} = m_*\bar{\theta} + \frac{m_* m_0^2}{2}\left(\frac{\tilde{d}_{u(d)} - \tilde{d}_{d(u)}}{m_{d(u)}} - \frac{\tilde{d}_s}{m_s}\right) \quad (15)$$

where we have neglected terms of $O(\tilde{d}_{(u,d)}/m_s)$, assuming an approximate proportionality of the CEDMs to the quark masses, i.e. $\tilde{d}_d/\tilde{d}_s \sim m_d/m_s \ll 1$.

With these results in hand, we now turn to optimization of the OPE expression (9). Recall that there are generally two motivated approaches for fixing the mixing parameter β: (1) at a local extremum; or (2) to minimize the effects of the continuum and higher dimensional operators. Since we are restricted here to only the first two orders in the OPE, we make use of the second option, as in [5], and set $\beta = 1$ to cancel the subleading infrared logarithmic terms, which are ambiguous due to the cutoff. This procedure mimics the original motivation for $\beta = -1$ in the CP-even case [16].

On the phenomenological side of the sum rule we have

$$\Pi^{\text{phen}} = \frac{i}{2}\{F\sigma\gamma_5, \not{p}\}\left(\frac{\lambda^2 d_n m_n}{(p^2 - m_n^2)^2} + \frac{A}{p^2 - m_n^2}\cdots\right). \quad (16)$$

We retain here the double and single pole contributions, the latter corresponding to transitions between the neutron and excited states, but the exponentially suppressed continuum contribution will be ignored. In (16) $\lambda = \lambda_1 + \beta\lambda_2$ and A is an effective constant parametrizing the single pole contributions. A more detailed analysis will be presented elsewhere [12], and we have verified that the inclusion of a continuum does not affect the analysis below.

After a Borel transform of (9) and (16), and using $\beta = 1$ as discussed earlier, we obtain the sum rule,

$$\lambda^2 m_n d_n + AM^2 = -\frac{M^4}{32\pi^2}e^{m_n^2/M^2}\langle\overline{q}q\rangle$$
$$\times\left[\pi_{\beta=1}^{(\chi)} + \pi_{\beta=1}^{(\xi,\kappa)} + \pi_{\beta=1}^q\right] + O(M^2), \quad (17)$$

where the contributions now take the elegant form,

$$\pi_{\beta=1}^{(\chi)} = 4\left[4\chi_d m_d P_d - \chi_u m_u P_u\right] \quad (18)$$

$$\pi_{\beta=1}^{(\xi,\kappa)} = \frac{1}{2}\left[4\tilde{d}_d\alpha_d^+ - \tilde{d}_u\alpha_u^+\right] \quad (19)$$

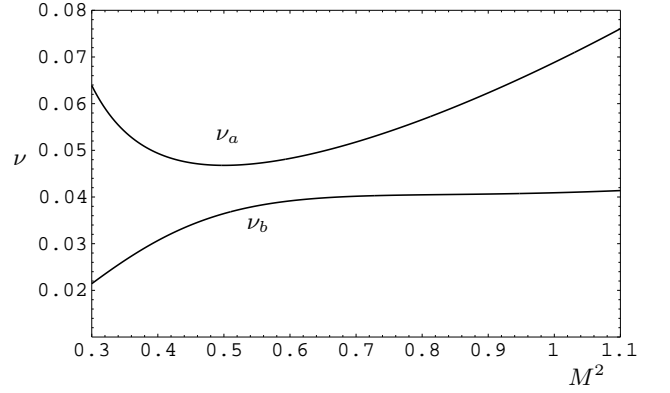$$\pi_{\beta=1}^q = 4\left[4d_d - d_u\right]. \quad (20)$$

FIG. 2: The neutron EDM function $\nu(M^2(\text{GeV}^2))$ is plotted according to the sum rules (a) and (b).

It is remarkable that the contribution of the u quark is $-1/4$ that of the d quark, which is precisely the combination suggested by the SU(6) quark model!

In order to analyze the sum rule (17), we first determine the coupling λ using the sum rules for the tensor structures $\mathbf{1}$ and $\not{p}$ in the CP even sector (see e.g. [17] for a recent review). Following [5], we construct two sum rules. Firstly, (a): we extract a numerical value for λ via a direct analysis of the CP even sum rules. This analysis has been discussed before and will not be reproduced here (see e.g. [17]). One uses $\beta = -1$, and obtains $(2\pi)^4\lambda \sim 1.05 \pm 0.1$. As an alternative, (b): we extract λ explicitly as a function of β from the CP-even sum rule for $\not{p}$, and substitute the result into (17) choosing $\beta = 1$.

The conventional approach which we shall adopt here is to assume that A is independent of M, and thus the left hand side of (17) is linear in M^2 provided that λ is constant in the appropriate region for the Borel parameter. For case (b), the latter point may be verified explicitly. The function $\nu(M^2)$, given by

$$\nu(M^2) \equiv \frac{1}{2[\pi^{(\chi)} + \pi^{(\xi,\kappa)} + \pi^q]}\left(d_n + \frac{AM^2}{\lambda^2 m_n}\right), \quad (21)$$

is then determined by the right hand side of (17).

The two sum rules described above for ν_a and ν_b are plotted in Fig. 2. $\nu(M^2)$ is to be interpreted as a tangent to the curves in Fig. 2. For numerical calculation we make use of the following parameter values: For the quark condensate, we take $\langle\overline{q}q\rangle = -(0.225 \text{ GeV})^3$, while for the condensate susceptibilities, we have the values $m_0^2 = 0.8 \text{ GeV}^2$ [18], $\chi = -5.7 \pm 0.6 \text{ GeV}^{-2}$ [18], $\xi = -0.74 \pm 0.2$ [8], and $\kappa = -0.34 \pm 0.1$ [18].

One observes that both sum rules have consistent extrema suggesting that our procedure for fixing the parameter β is appropriate. Furthermore, the differing behavior away from the extrema implies that for consistency we must assume A to be small. One then finds d_n as given by $\nu(M^2 \sim 0.55\text{GeV}^2)$. This low scale is characteristic of CP violating effects, and convergence of the OPE is apparently not in danger due to the combinatoric suppression factors which arise in extracting the CP violating

sources from higher dimension operators.

Extracting a numerical estimate for $\nu(M^2 \sim 0.55\text{GeV}^2) \sim 0.043$ from Fig. 2, and determining an approximate error due to higher order corrections and the sum rule analysis, we find the result[20]:

$$
\begin{aligned}
d_n &= (0.4 \pm 0.2) \left[\chi(4e_d - e_u)\left(m_*\bar{\theta} - \frac{1}{2}m_0^2\tilde{d}_s\frac{m_*}{m_s}\right) \right. \\
&+ \frac{1}{2}\chi m_0^2(\tilde{d}_d - \tilde{d}_u)\frac{4e_d m_d + e_u m_u}{m_u + m_d} \\
&+ \left. \frac{1}{8}(4\tilde{d}_d\alpha_d^+ - \tilde{d}_u\alpha_u^+) + (4d_d - d_u) \right], \quad (22)
\end{aligned}
$$

for the neutron EDM. The CEDM contributions are significant and comparable in magnitude in fact to the effects induced by quark EDMs.

For most phenomenological applications, it is necessary to invoke a PQ symmetry, which removes the dominant contribution $d_n(\bar{\theta})$. However, while setting $d_n(\bar{\theta}) = 0$, this symmetry induces additional CP violating terms through linear contributions to the axion potential [10]. In particular, the axion potential has the form $V \sim -\theta^2 K - 2\theta K'$, where $K = i(\alpha_s/8\pi)^2\langle G\tilde{G}, G\tilde{G}\rangle$ is the topological susceptibility, and $K' = i(\alpha_s/8\pi)\langle G\tilde{G}, \delta\mathcal{L}^{\text{CEDM}}\rangle$ are correlators arising from the CEDM sources. This linear shift in the axion potential then leads to an "induced" θ-term with coefficient (see e.g. [15])

$$
\theta_{\text{ind}} = -\frac{K'}{K} = \frac{m_0^2}{2}\sum_{q=u,d,s}\frac{\tilde{d}_q}{m_q}, \quad (23)
$$

which importantly is independent of any of the specific details of the axion mechanism.

Including these contributions, we observe the complete cancellation of the term proportional to the strange quark CEDM, and (22) takes a remarkably simple form

$$
\begin{aligned}
d_n^{\text{PQ}} &= (0.4 \pm 0.2)\left[4d_d - d_u + \frac{1}{2}\chi m_0^2(4e_d\tilde{d}_d - e_u\tilde{d}_u) \right. \\
&+ \left. \frac{1}{8}(4\tilde{d}_d\alpha_d^+ - \tilde{d}_u\alpha_u^+)\right]. \quad (24)
\end{aligned}
$$

Substituting numerical values for the condensates, we obtain

$$
\begin{aligned}
d_n^{\text{PQ}} &= (1 \pm 0.5)\frac{|\langle\bar{q}q\rangle|}{(225\text{MeV})^3} \\
&\times \left[1.1e(\tilde{d}_d + 0.5\tilde{d}_u) + 1.4(d_d - 0.25d_u)\right]. \quad (25)
\end{aligned}
$$

The result for $d_n^{\text{EDM}}(d_q)$ agrees well with lattice calculations of quark tensor charges of the proton [19], and with the naive quark model estimate. Note that an overall factor of $\langle\bar{q}q\rangle$ combines with the light quark masses from short-distance expressions for d_i and $\tilde{d}_i$ to give a result

$\sim f_\pi^2 m_\pi^2(1 + O(m_{(u,d)}))$ thus reducing the uncertainty due to poor knowledge of the quark masses and condensates. Further progress in calculating d_n would require more elaborate analysis of the sum rules in order to reduce the error in the overall coefficient, while the relative coefficients of different terms in the square brackets of (25) are likely to remain the same.

In conclusion, we have presented the first systematic study of the neutron EDM induced by all CP violating sources of dimension four and five within QCD sum rules. We observe that the contributions from the CEDM sources, including that of the strange quark, are large – in fact as large as those associated with the quark EDM sources. This result has significant implications for the analysis of the EDM constraints imposed on the supersymmetric and other models of CP violation.

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